The Early Career Section offers information and suggestions for graduate students, job seekers, early career academics of all types, and those who mentor them. Angela Gibney serves as the editor of this section assisted by the Early Career Intern Katie Storey. Next month's theme will be advice from the BIG Math Network. For all the Early Career articles that have appeared so far listed by topic, see `www.angelagibney.org/the-ec-by-topic`.

In Celebration of Women's History Month

Advice from our Advisor: Fan Chung

Sinan G. Aksoy

Fan and the World of Mathematics

As I reached for my wallet to offer my share for dinner, my advisor stopped me and said, "Listen, Sinan. We're going to have a lot of dinners together here. Let's just decide now that I'm going to pay for every single one of them. Your advisor pays for dinner. Once you have students of your own, then it will be your turn. OK?" Thanking her, I put my wallet away. I began to walk back toward my hotel and Fan said "Meet you here again on Wednesday?"

We had just landed in Taipei for a semester-long visit at National Taiwan University. In the following months, I would complete the first major result of my thesis. Fan and I would pour over dozens of drafts of our first joint paper. She would help break my habit of writing overly dense sentences (*"If you try to say too much all at once, you'll end up saying nothing at all!"*), impart the importance of a compelling and succinct introduction (*"You know, many people won't read your paper beyond the intro and theorem statement..."*), and push me to strengthen our main result, even when I was certain we were finally done (*"It's normal to have dozens of 'final drafts' before actually converging!"*). Through all this, I'd begin to feel a sense of security and optimism about my prospects as a career mathematician. But before all that, my advisor was establishing a different type of routine for this chapter of our collaboration: she was making sure that, in a place where I knew no one else, I'd have a friend with whom to regularly eat dinner.

These are the types of interactions that come to mind when I reflect on what my advisor taught me. In this article, I and Fan's other students recount lessons we learned from her, organizing them into three "axioms" for flourishing in the mathematical world. As will soon be made clear, Fan Chung showed us the role of a good advisor was not limited to the classroom. Sure, she would walk you through the spectral proof of Szemerédi's Regularity Lemma. But

Sinan G. Aksoy is a scientist at Pacific Northwest National Laboratory. His email address is `sinan.aksoy@pnnl.gov`.

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she would also counsel you on how to handle aggressive questions during math talks, how to rebound when another researcher publishes the theorem you were working on before you, and how to navigate the social complexities that arise in collaboration. All the while, she was your friend. Perhaps underlying Fan's advising style is the recognition that a myriad of sometimes-fragile things must happen to grow and sustain a healthy career as a mathematician. Fan's best advice was given, often by example, on how to thrive within the *world of mathematics*. Our accounts of her "advising axioms" are by no means comprehensive, since, as put by Josh Cooper:

> Fan Chung has been a tremendous source of inspiration for generations of mathematicians working in so many areas it is hard to even categorize them. In addition, she is a font of profoundly good advice. Fellow graduate students and I never knew quite what to think about Fan's penchant for waxing philosophical in class, in discussions in her office, and in research talks. Much to my surprise, these moments of casual commentary and impressionistic meta-cognition about mathematics have stuck with me over the years and have become integral to my own research advisement! I hear myself repeat her words to students frequently, and I often find myself wishing that I followed her advice more closely myself.

Indeed, for many of us, both in our recollections and through our continued interactions with her, Fan's guidance is ongoing: "Even now, more than a decade after finishing my degree, Fan is still interested in hearing what I am doing and offers suggestions on different opportunities I should explore," says Steve Butler. "For Fan, being an advisor to her students is more than getting a dissertation completed, it involves setting students up for long-term success and is a lifetime commitment." As a first step towards this success, Fan acclimated her students to the sometimes-dizzying and failure-prone nature of mathematics research, via a problem selection approach we dub the "velcro ball method."

Axiom 1: Use the Velcro Ball Research Method

On Fan's bookshelf, a plastic desk sign tucked in the corner reads:

> *A creative mess is better than idle tidiness.*

Sometimes overwhelmed, I found myself staring at this sign—the words "creative mess" glaring—when listening to Fan launch into *yet another* open problem during our weekly meeting. As noted by several of Fan's collaborators and students, Fan advocated working on multiple problems simultaneously. As put by past student Franklin Kenter:

> Fan was a proponent of the "velcro ball method." Instead of dictating what problem to work on or what direction to take, Fan would expose her students to a multitude of different problems. During individual research meetings, the students take the reins and discuss their progress. If they "forget" to bring up a certain point or problem, then the "velcro ball" didn't stick. Time to throw another!

The velcro ball method was not without an initial adjustment period. Sam Spiro recounts:

> Almost every time I met with her, Fan would disregard whatever we were working on last week, and instead give me a completely new problem to work on. At first I didn't make progress on any of these problems, which made me feel like I was failing Fan. Eventually I realized that Fan was by no means expecting me to solve all of these problems: if I could solve one in ten of them, I would be making good progress. And indeed, before long, she threw out a problem that I actually made some progress on, and eventually I ended up solving it. After this happened a few more times, I really began to appreciate Fan's methodology.

Many of Fan's former students report using Fan's frenzied approach to problem selection. As a practical matter, sometimes retreating from a problem gives us much needed headspace: "That way, if you get stuck, there is something else to think about to allow your subconscious to chew on the sticking point," notes Josh Cooper. As a tool for teaching, Franklin Kenter found her approach elicits introspection within his students: "Currently, I use this approach with my undergraduate students; albeit, with more guidance. Ultimately, a student project is what they make of it, and it is best if the student has interest and takes ownership in the specific topic." In my own case, I've internalized Fan's velcro ball method as a norm for maintaining productivity in a multidisciplinary research environment. For national laboratory scientists like myself, pivoting between multiple projects on a daily basis is a necessity. In a setting where I may interact with chemists, biologists, and power grid engineers within a single day, Fan's training keeps me comfortable and grounded in what might otherwise feel like a chaotic environment.

Axiom 2: The Math Career Graph Is an Expander—Find Your Path!

Just as the velcro ball method prompted us to sift through options and reflect on the problems *we liked*, Fan encouraged us to reflect more broadly on rich options available to us as early-career mathematicians. With her own varied career spanning academia and Bell Labs serving as proof, Fan sought to empower us with frequent reminders that mathematicians are uniquely positioned to pursue a plethora of career paths: finance, government, industry,

teaching, academia, and more. To be clear, Fan didn't feel her job was to steer you towards any one of these—or any subfield within mathematics for that matter. Ross Richardson recounts:

> Despite being the driver of a number of distinct research programs, in my tenure I never observed Fan to push any of her students into a particular line of research or force them to advance or advocate for her projects. She unflappably supported me in each direction I pursued. While she is available to students, she understands the value of allowing students to motivate and pace themselves. Her active role as advisor is one of connecting and empowering her students.

Instead of choosing your path for you, Fan instead asked you to start by committing (firmly, for now) to a path. Once you had, Fan assuaged graduation fears by acknowledging she had no intention of getting in your way. Franklin Kenter recalls:

> "Find a job, and the thesis will write itself" was something Fan would often repeat. The point here is that Fan was very mindful of her students' career goals and provided opportunities for her students to see those career goals. Not every student has the same desires; some want to teach, some want to work in tech, others want to work in national labs, and so on. Each of these directions requires a slightly different emphasis, but they all require research experience nonetheless. Typically, if one had enough research experience to secure *their* job, then they had enough for a thesis!

Franklin describes his appreciation for this attitude, having been both on the giving and receiving end:

> As an undergraduate project advisor, I take this philosophy to heart. For instance, a student once informed me they were going to focus on studying for their professional exams instead of working on our project for one whole month. I could have been annoyed or even threatened their grade, but in the end I should be as supportive of their goal as possible. They passed with flying colors, and it made the final months much more pleasant as an advisor—even resulting in a submission. Indeed, "they found their job, and the thesis wrote itself."

Lastly, Fan's career advice also acknowledged the broader social contexts in which we work. As one example of such, Olivia Simpson reflects on how Fan made a name for herself as a female mathematician while also working at the same institutions as another renowned mathematician, her husband Ron Graham:

> I think she also understood that the playing field was a little different for me as a woman. I'm not sure she ever explicitly expressed this to me, but I felt a nurturing from her that went beyond what I thought an advisor relationship would be. I remember once asking her what it was like starting out, working with Ron at Bell Labs, and she said that it was "quite hard to not be in his shadow." This really helped me to understand that she helped pave an important path for women in mathematics.

It is doubtless Fan's role as a trailblazing female mathematician—including being among an early group of women to get tenure from Ivy League universities—paved a path for many women in mathematics. Her success also underscores her simple advice to young women interested in math: *don't be intimidated!* Not only does a confident mindset overcome the all-too-common fear barrier in mathematics, but it also puts us at ease during an activity Fan cherished: collaborating with others.

Axiom 3: Collaboration Means Family

The first time I heard Fan say, "You know, collaboration is a much closer relationship than most ordinary friendships," I didn't really get it. Over the years, I've come to appreciate how far this viewpoint is from hyperbole. Much of Fan's advice stressed the importance of, as put by Ross Richardson, "how we communicate, promote, and socialize our work as part of a delicate series of social interactions." Continuing, Ross writes:

> If there is one point she makes explicitly more than any other, it is the importance of the social aspect of mathematics, and the intense and joyful friendships that come from collaboration. This emphasis is unnecessary, however—it is clear in how much energy she puts into her

Figure 1. Fan Chung, Ron Graham, and many of their past students at the Networked Life conference in 2016.

> collaborations that they are the highlight of her mathematical life.

To Fan, collaboration means wholly committing your time and energy to your collaborators. Fan was available at seemingly any hour of the day; "an emailed question on a Friday night could lead to an hour-long conversation on Saturday morning," recalls former student Josh Tobin. Above all, Fan advocated for garnering trust with collaborators by behaving *generously*. For example, Richardson recalls Fan's willingness to share credit:

> I remember her handing me a draft latex file containing an unfinished attack on the Erdős unit distance problem. It was still raw and contained new unpublished ideas, but she gave it to me freely based on my interests to see if I could push it forward. I was floored that she would hand it over, and it stayed with me as a mark of the respect and trust she had in her students and in what she expected to be part of any collaboration.

As put by Alex Tsiatas, this intellectual generosity is a reflection of the fact that "Fan practiced research without ego." Another facet of this generosity is acknowledging the contributions of others. Distilled to three words, Josh Cooper summarizes Fan's advice in this regard as: "*Don't burn bridges!* Math is about community, and so requires care in addressing colleagues. It costs nothing to include another coauthor, but not doing so can lead to all kinds of headaches. In fact, after a paper is written, it is wise to take the stance that each person involved probably did 75% of the work." As my own network of collaborators has grown, I increasingly take Fan's attitude towards collaboration as essential. The undertaking of any collaboration requires granting trust and showing intellectual vulnerability: trust that everyone will see through the (often years-long, arduous) process of paper drafting, submission, referee reports, and revisions, trust that everyone will maintain mutual respect should they make mistakes or contribute significantly more or less than their coauthors, and trust in our ability to effectively co-steward and gracefully share credit for ideas which form the basis of our careers. I view Fan's generosity practices as ways of honoring these types of trust.

Through these practices, Fan advocated cultivating a family of collaborators throughout one's career. For Fan, this sometimes manifested in the near-literal sense of the word "family." Steve Butler realized this early in graduate school:

> When I was in graduate school I moved a significant distance from San Diego because of my wife's work. So I started sleeping in my office several nights a week to limit my commuting. Eventually word of this got to my advisor (Fan) and instead of ignoring the situation or kicking me out of my office, Fan offered up a room in her basement that I could stay in when I needed. Over the course of the next several years I spent hundreds of nights there, and became like a part of the family.

In fact, a number of Fan's students feel familial bonds with her. "She considers me and my family as a part of her extended family," says Linyuan Lu, fondly recalling how "she gave gifts to my children and would take us to have Dim Sum with her at San Diego's Emerald Restaurant." Similarly, Mark Kempton was struck by how "Fan always took an interest in my family after I got married and had kids." I too feel the same bond with Fan.

Nevertheless, as family, collaborators must also be willing to show tough love when necessary. "Fan didn't hold back with her criticism," recalls Mark Kempton. Linyuan Lu similarly echoes, "When something goes wrong, she is not afraid of pointing out my errors!" In particular, Fan's students quickly become acutely aware of her uncompromising standards for the tone, flow, and presentation of math talks. Paul Horn recounts a common experience of tough love:

> I thought highly of my talks at the time and so it was surprising to me when, after the talk, everyone left the room except for Fan and Jeff Remmel and she absolutely obliterated my talk. She deconstructed it from beginning to end, pointing out myriad mistakes. I was stung—I thought I had done well. But, as I processed, I also realized she was absolutely right on every point. That day, I started to completely rethink how I plotted out my talks and slides and today, whatever flaws my talks may have, they are 1000 times better thanks to her.

Linyuan Lu similarly reports benefiting from Fan's criticism of his job talk, crediting his job offer at the University of South Carolina in 2004 to the "weeks of training and practice Fan and Ron's mom provided on pronunciation and presentation."

In conclusion, Fan Chung's advising axioms are aimed at protecting and nourishing the joy we share by engaging in mathematics together. From her problem solving approach, to her career guidance, to her collaboration ethics and beyond, her advising has profoundly impacted her students' careers, affecting our research, teaching, and management practices in diverse and substantive ways. Whether it be Paul Horn emulating the "pride she took in our accomplishments and opportunities she afforded her graduate students," Steve Butler passing on "Fan's compassion and caring," or Jake Hughes and Alex Tsiatas applying her collaboration philosophies to "foster an inclusive, diverse, high-performing team" in industry settings, we are in agreement with Josh Cooper when he says: "Fan's thoughtful influence on her students and colleagues will

have as lasting an impact on mathematics as her theorems." I submit there is much to be gained by internalizing her axioms, for students, educators, and all those inhabiting the mathematical world alike.

Sinan G. Aksoy

Credits

Figure 1 is courtesy of Todd Kemp.
Photo of Sinan G. Aksoy is courtesy of Sinan G. Aksoy.

Celebrating Karen Parshall as an Advisor

Karen Parshall is the Commonwealth Professor of Mathematics and History at the University of Virginia. Her extensive research focuses on the history of nineteenth- and twentieth-century mathematics. She was named an inaugural Fellow of the American Mathematical Society in 2012 and a Fellow of the American Association for the Advancement of Science in the Section on Mathematics in 2020. In 2018, she received the Albert Leon Whiteman Memorial Prize of the American Mathematical Society "for her outstanding work in the history of mathematics, and in particular, for her work on the evolution of mathematics in the United States and on the history of algebra, as well as for her substantial contribution to the international life of her discipline through students, editorial work, and conferences."[1] Here, her graduate students Della Dumbaugh, Patti Hunter, Sloan Despeaux, Deborah Kent, and Laura Martini (organized in order of completion of their PhD) offer reflections on their experiences while working with Karen Parshall.

[1]*The citation for this prize offers an overview of Parshall's scholarly work along with biographical and autobiographical insights. See "2018 Albert Leon Whiteman Prize Announcement,"* Notices of the American Mathematical Society *65 (4), 2018, 472–474.* https://www.ams.org/journals/notices/201804/rnoti-p472.pdf.

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Della Dumbaugh

When I arrived at the University of Virginia as a graduate student in the fall of 1988, I planned to study pure mathematics, with some combination of algebra and number theory. One rainy Friday evening that first semester I stumbled on a copy of Carl Boyer's *A History of Mathematics* at the local independent bookstore in Charlottesville. I couldn't put it down that weekend. I decided to take a class that spring in the history of mathematics. This meant listening to Karen Parshall talk about the history of calculus, learning about and reading primary sources, and writing papers on a broad range of topics, including the history of the solution to the cubic. I was hooked.

This class led to a joint project with Karen exploring the American mathematical community as it took shape in the late nineteenth- and early twentieth-centuries. This entailed me driving to Karen's lovely home on Coleman Drive, to the sunny solarium where we combed through early editions of the *Bulletin of the New York* and, later, *American Mathematical Society* and meticulously recorded information about mathematicians, talks, conferences, and institutions. These details ultimately combined to identify a growing, vibrant community of mathematicians. As we worked, we talked about our observations. Who was this Leonard Dickson who kept giving talks and writing papers? Why were there so few women recorded on these early pages of the *Bulletin*? What was going on at the University of Chicago? I listened to Karen talk more broadly about these observations and queries that arose along the way. Looking back, these long afternoons form some of my most treasured moments with Karen. I had the chance to hear her think out loud about the early American mathematical community as new ideas unfolded before us.

Sometime later, I made an appointment with Karen, drove out to Coleman Drive, handed her a single piece of paper with an outline of a dissertation on Leonard Dickson and his work in the theory of algebras, and asked her if she would take me on as a student. She said yes. That moment, I suppose, was the beginning of my work with her as a graduate advisor. Through Karen's expert guidance, that piece of paper eventually grew into a 237-page dissertation. How did that happen? That evolution hinged on what I consider the two sterling features of Karen Parshall as PhD advisor: her commitment to weekly meetings and her focus on writing. The former taught consistency and the latter attention to detail. To this day, I am never far from my current research project and I am unafraid to print out my written work and take a red pen to it. But these skills did not come easily. Meeting with Karen to discuss Bruno Latour or Thomas Kuhn could leave me frazzled and

Della Dumbaugh is a professor of mathematics at the University of Richmond and editor of the American Mathematical Monthly. *Her email address is* ddumbaugh@richmond.edu.